

Holiday-Homework, October 2019
Maths - XI

Solve following Ques :

- ① In a survey of 200 students of a school, it was found that 120 study Maths, 90 study Physics and 70 study Chemistry, 40 study Maths and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Maths and 20 none of these subjects. Find the number of students who study all the three subjects.
- ② In a group of 60 people, 25 play Cricket and 20 play Tennis and 10 people play both the games. Find the number of students who play neither.
- ③ Find the domain and range of the functions:
 - (i) $f(x) = \frac{1}{\sqrt{x-5}}$, $x > 5$.
 - (ii) $f(x) = |x-11|$, $x \in \mathbb{R}$.
- ④ Prove that $\therefore \cot x \cdot \cot 2x - \cot 2x \cdot \cot 3x - \cot 3x \cdot \cot x = 1$.
- ⑤ Find the principal and general solutions of the following equations.
 - (i) $\sec^2 2x = 1 - \tan 2x$
 - (ii) $\sin x + \cos x = 1$.
- ⑥ If $\cos(\theta + \phi) = m \cos(\theta - \phi)$, then prove that $\tan \theta = \frac{1-m}{1+m} \cdot \cot \phi$.
- ⑦ If $\tan \theta + \tan 2\theta + \sqrt{3} \tan \theta \cdot \tan 2\theta = \sqrt{3}$, then show that $\theta = \frac{n\pi}{3} + \frac{\pi}{9}$.
- ⑧ Prove the following by using the PMI for all $n \in \mathbb{N}$.
 - (i) $41^n - 14^n$ is a multiple of 27.

(ii) $1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \dots + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$.

(iii) $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$, for all natural numbers n .

9 Convert the following into polar form:

(i) $\frac{1+7i}{(2-i)^2}$ (ii) $\frac{1+3i}{1-2i}$

10 Find the modulus and argument of the complex number

$$z = \frac{1+2i}{1-3i}$$

11 If α, β are different complex numbers with $|\beta| = 1$, then find $|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}|$.

12 Show that the complex number z , satisfying the condition, $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$, lies on a circle.

13 Solve the following inequalities graphically -

(i) $3x+4y \leq 60, x+3y \leq 30, x \geq 0, y \geq 0$

(ii) $3x+2y \leq 150, x+4y \leq 80, x \leq 15, y \geq 0, x \geq 0$.

14 The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm, then find the minimum length of the shortest side.

15 We wish to select 6 persons from 8 but if the person A is chosen, then B must be chosen. In how many ways can selections be made?

16 A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5, without repetitions. Find the total no. of ways, in which this can be done.