

Holiday - Homework, October 2019

Maths - XII

Solve following questions.

- ① Show that the function f in $A = \mathbb{R} - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1} . [CBSE 2012].
- ② Show that the relation R in the set $N \times N$ defined by $(a, b) R (c, d)$, if $a^2 + d^2 = b^2 + c^2, \forall a, b, c, d \in N$ is an equivalence relation. [CBSE 2009].
- ③ If $f, g: \mathbb{R} \rightarrow \mathbb{R}$ are two functions defined as $f(x) = |x| + x$, and $g(x) = |x| - x, \forall x \in \mathbb{R}$. Then find $f \circ g$ and $g \circ f$. [CBSE 2014].
- ④ (i) Write the principal value of $\cos^{-1}(\cos 68^\circ)$. [CBSE 2012, 2014].
(ii) Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$. [CBSE 2011, 2014].
- ⑤ Solve for x : $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{9}{31}\right)$. [CBSE 2018]
- ⑥ Show that: $2 \tan^{-1}\left\{\tan \frac{\alpha}{2} \cdot \tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right\} = \tan^{-1}\left\{\frac{\sin \alpha \cdot \cos \beta}{\cos \alpha + \sin \beta}\right\}$. [CBSE 2011].
- ⑦ Prove that: $\tan\left\{\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{a}{b}\right\} = \frac{2b}{a}$. [CBSE 2017].
- ⑧ If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1} , where I and O have their usual meanings. [CBSE 2011, 2015].
- ⑨ (i) If A is an invertible matrix of Order 2 and $\det(A) = 4$, then write the value of $\det(A^{-1})$. [CBSE 2017].
(ii) If for any 2×2 square matrix A , $A(\text{adj. } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$. [CBSE 2017].
- ⑩ Show that the inverse of a matrix, if it exists, is unique. [CBSE 2011].

- (11) Show that $f(x) = x^2$, is differentiable at $x=2$.
- (12) Differentiate $\sqrt{\tan x}$ w.r.t. x . [CBSE 2009].
- (13) If $f(x) = \begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2}, & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous at $x=2$. Find the value of k . [CBSE 2009].
- (14) Let $f(x) = x \cdot |x|$, for all $x \in \mathbb{R}$. Discuss the continuity of $f(x)$ at $x=0$. [CBSE 2016].
- (15) If Rolle's theorem is applied on $f(x) = x^2 + 2$, in the interval $[-2, 4]$. Then what should be the value of c . [CBSE 2013, 2016].
- (16) For what values of λ , the function defined by -
 $f(x) = \begin{cases} \lambda(x^2+2), & \text{if } x \leq 0 \\ 4x+6, & \text{if } x > 0 \end{cases}$ is continuous at $x=0$?
 Hence check the differentiability of $f(x)$ at $x=0$. [CBSE 2015].
- (17) AB is a diameter of a circle and C is any point on the circle. Show that the area of ΔABC is maximum, when it is isosceles. [CBSE 2014 (C)].
- (18) Evaluate $\int_1^3 (2x^2 + 5x) dx$ as a limit of sum. [CBSE 2012].
- (19) Evaluate $\int_0^{\pi/2} (2 \log |\sin x| - \log |\sin 2x|) dx$. [CBSE 2009, 2013].
- (20) Evaluate : (i) $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$
 (ii) $\int_0^{\pi} \left(\frac{x \cdot \tan x}{\sec x + \tan x} \right) dx$. [CBSE 2010, 2014, 2017].

Note :- Solve the sample papers issued by CBSE (2019-20).

